

Math Skills for Chemistry 210

High school algebra is a requirement for Chemistry 210. For the most part lecture time is devoted to the topics of chemistry and not to review of these math skills. Many of you may have difficulty recalling the math concepts that are necessary to do chemistry problems. Here are some reminders about basic skills. Practice problems are provided. You should be able to solve any of these problems relatively quickly.

Algebra—solve an equation with one unknown

(1) Multiply or divide both sides of the equation by the same quantity:

$$\begin{array}{ll} 10 = \frac{20}{x} & \text{Multiply each side by } x \text{ to get ...} \\ 10x = 20 & \text{Divide each side by } 10 \text{ to get ...} \\ x = 20/10 = 2 & \end{array}$$

Ex.: In a first order process, if $t_{1/2} = 163$ minutes, what is k ?

$$\begin{array}{ll} t_{1/2} = \frac{0.693}{k} & \text{Multiply each side by } k \text{ to get ...} \\ (k)t_{1/2} = 0.693 & \text{Divide each side by } t_{1/2} \text{ to get ...} \\ k = \frac{0.693}{163min} & k = 0.00425min^{-1} \quad \text{Note } min^{-1} = 1min \end{array}$$

(2) Add or subtract both sides of an equation by the same quantity:

$$\begin{array}{ll} 5x - 8 = 3x + 12 & \text{Add } 8 \text{ to both sides to get ...} \\ 5x = 3x + 20 & \text{Subtract } 3x \text{ from both sides to get ...} \\ 2x = 20 & \text{Divide each side by } 2 \text{ to get ...} \\ x = \frac{20}{2} = 10 & \end{array}$$

Ex.: For a second order process, if $[A]_0 = 4.46M$, $[A] = 1.12M$, and $k = 0.045hrs^{-1}M^{-1}$, calculate t .

$$\begin{array}{ll} \frac{1}{[A]} = kt + \frac{1}{[A]_0} & \text{Subtract } \frac{1}{[A]_0} \text{ from both sides to get ...} \\ \frac{1}{[A]} - \frac{1}{[A]_0} = kt & \text{Divide each side by } k \text{ to get ...} \\ \left(\frac{1}{[A]} - \frac{1}{[A]_0}\right)/k = t & \text{Substitute in the values to get ...} \\ \left(\frac{1}{1.12M} - \frac{1}{4.46M}\right)/0.045hrs^{-1}M^{-1} = t \\ (0.893M^{-1} - 0.224M^{-1})/0.045hrs^{-1}M^{-1} = t \\ 0.669M^{-1}/0.045hrs^{-1}M^{-1} = t = 14.9hrs \end{array}$$

Note M^{-1} cancel, leaving $14.9/hrs^{-1}$. Since $1hrs^{-1} = 1hrs$, the units for the answer are hrs . Always check the units to make sure that they give what you are looking for.

(3) The Quadratic Formula

In case you have forgotten, the quadratic formula can be used to solve a quadratic equation. First, put the equation in the form $ax^2 + bx + c = 0$

Identify a, b, c.

Use the quadratic formula to get the values of x: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here is a simple example. Solve for x.

$$\frac{x^2}{1-x} = 4 \quad \text{Multiply both sides by (1-x) to get...}$$

$$x^2 = 4(1-x) \quad \text{Expand out the right side to get...}$$

$$x^2 = 4 - 4x \quad \text{Add 4x to both sides to get...}$$

$$x^2 + 4x = 4 \quad \text{Subtract 4 from both sides to get ...}$$

$$x^2 + 4x - 4 = 0$$

Now you have the equation in the form of $ax^2 + bx + c = 0$ where $a = 1$, $b = 4$, $c = -4$

Plug these into the quadratic formula to get the two solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-4)}}{2(1)} \quad x = \frac{-4 - \sqrt{4^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-4 + \sqrt{32}}{2} = 0.83 \quad x = \frac{-4 - \sqrt{32}}{2} = -4.83$$

Ex: If equilibrium concentrations are $[\text{COCl}_2] = 0.0100 - x$, $[\text{Cl}_2] = [\text{CO}] = 0.00100 + x$ and the equilibrium constant expression is $\frac{[\text{COCl}_2]}{[\text{Cl}_2][\text{CO}]} = 1.2 \times 10^3$, find x.

$$\frac{[\text{COCl}_2]}{[\text{Cl}_2][\text{CO}]} = 1.2 \times 10^3 \quad \text{Put in all the knowns to get ...}$$

$$\frac{0.0100 - x}{(0.00100 + x)(0.00100 + x)} = 1.2 \times 10^3 \quad \text{Multiply each side by } (0.00100 + x)^2 \text{ to get ...}$$

$$0.0100 - x = 1.2 \times 10^3 * (0.00100 + x)^2 \quad \text{Multiply out the right hand side to get ...}$$

$$0.0100 - x = (1.2 \times 10^3)x^2 + 2.4x + 1.2 \times 10^3 \quad \text{Put the equation in the form } ax^2 + bx + c = 0 \text{ to get...}$$

$$(1.2 \times 10^3)x^2 + 3.4x - 0.0088 = 0$$

$$\text{where } a = 1.2 \times 10^3, b = 3.4, c = -0.0088$$

The two solutions are:

$$x = \frac{-3.4 \pm \sqrt{3.4^2 - 4(1.2 \times 10^3)(-0.0088)}}{2(1.2 \times 10^3)}$$

$$x = \frac{-3.4 + \sqrt{53.8}}{2.4 \times 10^3} = 0.0016 \text{ or } x = -4.5 \times 10^{-3}$$

Note that the negative value of x does not make physical sense, only one solution: $x = 0.0016M$, is obtained.

Logarithms

A logarithm is the exponent to which you must raise the base to get a number. We use \log when the base is 10. We use \ln when the base is e . $e = 2.718\dots$

$$\log 10^x = x: \quad \log 10^3 = 3, \quad \log 10^{-5} = -5 \quad \log 10^{27} = 27$$

$$\ln e^x = x: \quad \ln 2 = 0.693, \quad \ln 10.00 = 2.303, \quad \ln 0.200 = -1.609$$

- To get the \log of a number, enter it into your calculator and press \log .
- To get the \ln of a number, enter it into your calculator and press \ln .
- You can convert from \log to \ln by this relationship: $\ln x = 2.303 \log x$

The anti-logarithm is the number corresponding to a given logarithm.

$$\text{antilog}(\log 10^x) = 10^x$$

$$\text{antiln}(\ln e^x) = e^x$$

In other words, the antilog function will remove the log term from an equation.

- To get the antlog of a number, enter the number and press inv log (when working in base 10) or inv ln (when working in base e).

To solve:

$$\ln x = 5$$

Take the antln of both side to get ...

$$x = \text{antiln}(5)$$

Push 5 and the inv ln button to get ...

$$x = 148$$

To solve:

$$\log x = 5$$

Take the antlog of both side to get ...

$$x = \text{antilog}(5)$$

Push 5 and the inv log button to get ...

$$x = 100,000 = 10^5$$

Ex.: If the initial concentration of sucrose is 0.010M, what is the concentration after 5.0 hours if the reaction is first order and $k = 0.21 \text{hrs}^{-1}$?

$$\ln \frac{[A]_0}{[A]} = kt \quad [A]_0 = 0.010M, k = 0.21 \text{hrs}^{-1}, t = 5.0 \text{hrs}$$

You must solve for $[A]$.

Put in all the knowns to get ...

$$\ln \frac{0.010M}{[A]} = (0.2 \text{hrs}^{-1})(5.0 \text{hrs})$$

Multiply out the right hand side to get ...

$$\ln \frac{0.010M}{[A]} = 1.05$$

Take the antiln of both sides to get ...

$$\frac{0.010M}{[A]} = \text{antiln}(1.05)$$

Solve the right hand side to get ...

$$\frac{0.010M}{[A]} = 2.86$$

Multiply each side by $[A]$ to get ...

$$0.010M = 2.86[A]$$

Divide each side by 2.86 to get ...

$$\frac{0.010M}{2.86} = [A]$$

Solve the left side to get ...

	Problem	Answer
1	$\frac{7y}{3} = 2y - 4$	$y = -12$
2	$4.0y = 6.0 \times 10^{-4}$	$y = 1.5 \times 10^{-4}$
3	$\frac{2x-8}{6} = 3 - 4x$	$x = 1$
4	$6.0y = (2.0 \times 10^{-2})(1.8 \times 10^3)$	$y = 6.0$
5	$(12.6)(0.0182) = y(1.4 \times 10^3)$	$y = 1.6 \times 10^7$
6	$y(1.6 \times 10^{-2}) = \frac{9.8 \times 10^4}{1.62}$	$y = 3.8 \times 10^6$
7	$\log 3.00$	0.477
8	$\log 4.00$	0.602
9	$\log 7.15$	0.854
10	$\text{antilog} 62$	10^{62}
11	$\text{antilog} 0.8641$	7.313
12	$\text{antilog} - 0.861$	0.1377
13	$\ln(2.02 \times 10^{-16})$	-36.138
14	$\ln(6.023)$	1.796
15	$\ln(2.718)^2$	2.000
16	$\ln(6.18 \times 10^{-5})$	-9.69
17	$y = -6.14 \times 100 \times \log 0.0360$	$y = 886$
18	$54.0 = \frac{116.2}{3.98} \log y$	$y = 70.7$
19	$-1.614 = (300y) \log(2.0 \times 10^3)$	$y = -1.6 \times 10^{-3}$
20	$\log 12 = \frac{0.019y}{2.30}$	$y = 1.3 \times 10^2$
21	$\log y = (1.2 \times 10^{-3})(3600)/2.30$	$y = 2.1$
22	$\log\left(\frac{3.0}{y}\right) = 0.16$	$y = 2.1$
23	$\frac{4x^2}{2-x} = 20$	$x = 1.5$ or -6.5
24	$\frac{3-x}{x^2} = 0.10$	$x = 2.4$ or -12.4
25	$\frac{x(0.50+x)}{1.00-x} = 0.010$	$x = 0.019$ or -0.51
26	$\frac{x^2}{1.0-x} = 0.30$	$x = 0.42$ or -0.72
27	$\frac{2}{(1.00-2x)^2} = 5.00$	$x = 0.365$ or 0.68